NUMERICAL MODELING OF LOCAL HEAT TRANSFER IN THE FURNACES OF TUBULAR OVENS, BASED ON DIFFERENTIAL APPROXIMATIONS FOR RADIANT HEAT TRANSFER

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The authors describe the calculation of the external heat transfer in the furnaces of tubular ovens of box type, based on numerical solution of systems of integro-differential equations of radiative gasdynamics using the K–E turbulance model and an empirical combustion model. Radiative heat transfer is considered in the S_2 approximation of the method of discrete ordinates, and the SIMPLE algorithm is used to solve the gasdynamic part of the problem. The influence of furnace width on the external heat transfer was investigated numerically.

Introduction. Tubular ovens are widely used in the oil-gas and oil technology industry. The main item most affecting the efficiency and reliability of operation of the tubular furnace is the radiant (furnace) chamber. In the furnace chamber of a tubular oven a number of interconnected physical and chemical processes occur: radiative and convective heat transfer, turbulent flow of combustion products, and combustion of fuel. Here an appreciable role is played by the location of the burner and the method of removing the combustion products from the furnace, by the circulation of combustion products, by the nature of the heat release in the jet volume, by the selectivity of the radiation, and by the other regime and structural parameters of the furnace.

At present the most developed method for thermal computation of furnaces is the zonal method [1, 2]. However, the difficulties arising when one matches the zonal approach to radiant heat transfer with a finite difference method of solving the equations of gasdynamics limit substantially the region of application of zonal computing methods. One direction where there has been improvement in methods of thermal design of furnace chambers is the development of differential methods, based on numerical solution of the system of integro-differential equations of radiative gasdynamics, and models of turbulence and combustion.

Mathematical Model. The tubular oven considered in this work, with a furnace of rectangular section, is characterized by a small width of radiant section compared with the length and height, and a symmetric location of the tubular screen and the series of burners (Fig. 1). In such furnaces the variation of flow parameters along the length is much less than it is over the width and height. Therefore, the problem of heat transfer and flow of combustion products may be examined in the two-dimensional formulation. Here the tubular screen is replaced by a light-sensitive surface, nontransparent to radiation, with an effective emissivity depending on the outer diameter, the emissivity of the tubes, and the pitch between them [3]. Because of the small diameter of the tubes compared with the size of the furnace chamber and the small pitch between the tubes, the tubular screen may be considered as a solid wall, which also simplifies the solution to the gasdynamic problem.

The radiant heat transfer is described using the method of discrete ordinates, according to which the radiative transfer equation is approximated by a system of differential equations for the radiative intensity along a specific number of chosen directions:

$$\mu_m \frac{\partial I_m^k}{\partial x} + \xi_m \frac{\partial I_m^k}{\partial y} = \frac{\alpha_{\lambda k}}{\Delta\lambda_k} \int\limits_{\lambda_{k-1}}^{\lambda_k} J_{\lambda b}(\lambda,\ T)\, d\lambda - (\alpha_{\lambda k} + \beta) I_m^k + \frac{\beta}{4\pi} \sum_{m'=1}^{N_0} w_{m'} I_{m'}^k. \qquad (1)$$

Each direction is assigned the angular coordinates $\{\mu_m,\ \xi_m;\ m = 1,\ N_0\}$, where m is the number of chosen directions.

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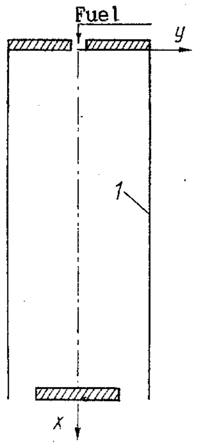

Fig. 1. Simplified model of a furnace and the coordinate system: I is the tubular screen.

The uniqueness condition for Eq. (1) at x = 0 for a diffusely radiating and reflecting wall is approximated as follows:

$$I_m^k = \frac{\varepsilon}{\Delta\lambda_k} \int\limits_{\lambda_{k-1}}^{\lambda_k} J_{\lambda b}(\lambda,\ T_w)\, d\lambda + \frac{r}{\pi} \sum_{m'=1}^{N_0} w_{m'} |\mu_{m'}|\, I_{m'}^k, \tag{2}$$

for values $\mu_m' < 0$ and $\mu_m > 0$. At the other boundaries there are uniqueness conditions analogous to Eq. (2). Our set of values $\{\mu_m, \xi_m;\ m = 1, N_0\}$ and weight factors w_m were borrowed from [4].

The local value of divergence of the radiative heat flux is computed from the formula

$$\operatorname{div} \mathbf{q}_p = \sum_{k=1}^{N_s} \alpha_{\lambda k} \left[\int\limits_{\lambda_{k-1}}^{\lambda_k} J_{\lambda b}(\lambda, T) d\lambda - \sum_{m=1}^{N_0} w_m I_m^k \Delta\lambda_k \right],$$

and the components of the vectorial radiative heat flux density integrated over the spectrum q_p are determined from the formulas

$$q_p^x = \sum_{k=1}^{N_s} \Delta\lambda_k \sum_{m=1}^{N_0} \mu_m w_m I_m^k\ ;\ q_p^y = \sum_{k=1}^{N_s} \Delta\lambda_k \sum_{m=1}^{N_0} \xi_m w_m I_m^k\ .$$

The temperature field required to solve Eq. (1) is determined by solving the energy conservation equation

$$c_p\rho u \frac{\partial T}{\partial x} + c_p\rho v \frac{\partial T}{\partial y} = \frac{\partial}{\partial x}\left[(\lambda + \lambda_\text{T}) \frac{\partial T}{\partial x}\right] + \frac{\partial T}{\partial y}\left[(\lambda + \lambda_\text{T}) \frac{\partial T}{\partial y}\right] + Q - \operatorname{div} \mathbf{q}_p. \tag{3}$$

It is postulated that the volume density of heat released Q varies only along the flame. Then the amount of heat Q_{1-2}, released between two sections of the flame $x = x_1$ and $x = x_2 (x_2 > x_1)$, can be evaluated from the formula

$$Q_{1-2} = B_\text{T} Q_\text{p}^{\ell} [\eta(x_2) - \eta(x_1)]. \tag{4}$$

The integral degree of combustion of fuel along the flame is given by the empirical dependence [5]

$$\eta(x) = 1 - \exp\left[-a\left(\frac{x}{l_f}\right)^2\right]. \tag{5}$$

The velocity field and the coefficients of turbulent transfer were determined by solving the time-averaged Navier–Stokes equations, the continuity equation, and the equation of the K–E turbulence model, which can be combined formally into the one general equation

$$\frac{\partial(\rho u\Phi)}{\partial x} + \frac{\partial(\rho v\Phi)}{\partial y} = \frac{\partial}{\partial x}\left(\Gamma_f \frac{\partial\Phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(\Gamma_f \frac{\partial\Phi}{\partial y}\right) + S_f, \tag{6}$$

where the general variable $\Phi = (u, v, 1, k, \varepsilon)$, and Γ_f and S_f are determined from Table 1.

TABLE 1. Expressions for Γ_f, S_f in the generalized Eq. (6).

Φ	Γ_f	S_f
u	$\mu+\mu_T$	$\frac{1}{3}\frac{\partial}{\partial x}\left(\mu_{ef}\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(\mu_{ef}\frac{\partial v}{\partial x}\right)-\frac{2}{3}\frac{\partial}{\partial x}\left(\mu_{ef}\times\frac{\partial v}{\partial y}\right)-\frac{\partial P}{\partial x}-\rho g\frac{T-T_0}{T_0}$
v	$\mu+\mu_T$	$\frac{1}{3}\frac{\partial}{\partial y}\left(\mu_{ef}\frac{\partial v}{\partial y}\right)+\frac{\partial}{\partial x}\left(\mu_{ef}\frac{\partial u}{\partial y}\right)-\frac{2}{3}\frac{\partial}{\partial y}\left(\mu_{ef}\times\frac{\partial u}{\partial x}\right)-\frac{\partial P}{\partial y}$
1	1	0
k	$\mu+\mu_T$	$G-\rho\varepsilon$
ε	$\mu+\frac{\mu_T}{\sigma_\varepsilon}$	$C_{\varepsilon 1}\frac{\varepsilon}{k}G-C_{\varepsilon 2}\frac{\rho\varepsilon^2}{k}$

Note: $G=\mu_T\left\{2\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2\right]+\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2\right\}$.

The furnace chambers of tubular ovens operate at relatively low pressures (on the order of 1 atm) and high temperatures (1200-1800 K), and therefore the state of the chimney gases can be described in the ideal gas approximation

$$P=\rho R_\mu T. \tag{7}$$

The turbulent viscosity and the coefficient of turbulent heat transfer are computed from the formulas

$$\mu_T=C_\mu f_{\mu 1}\rho\frac{k^2}{\varepsilon};\ \lambda_T=\frac{c_p\mu_T}{\mathrm{Pr}_T}, \tag{8}$$

where the function $f_{\mu 1}$ accounts for the influence of the turbulent Reynolds number Re_T on μ_T near the wall:

$$f_{\mu 1}=\exp\left[-\frac{a_3}{1+a_4\,\mathrm{Re}_T}\right].$$

The empirical constants in the turbulence model, following [6], were assumed to be: C_μ = 0.09; $C_{\varepsilon 1}$ = 1.44; $C_{\varepsilon 2}$ = 1.92; σ_ε = 1.3; Pr_T = 0.9; a_3 = 2.5; a_4 = 0.02.

To uniquely determine the solution of the system of equations (3) and (6) one must add the appropriate boundary conditions. The flow parameters at the entrance section are given, and therefore there a boundary condition of the first kind is applied. The kinetic energy and the turbulent dissipation at the chamber entrance are determined from the relations [7]

$$k_0=1{,}5\,u_0^2\,\mathrm{Tu};\quad \varepsilon_0=\frac{C_\mu^{0,75}k_0^{1,5}}{0{,}1D}.$$

At the exit section we use the method of "one-sided" coordinates [8], allowing us to close the original differential equations. On the heater surface the temperature of the external wall of the radiant tubes is given. The claque temperature is determined by solving the heat balance equation representing the resultant radiative heat fluxes, the convective heat fluxes and the thermal losses through the claque:

$$\frac{T_{\text{вн}}-T_w}{R_f}=[(\mathbf{q}_p\mathbf{n})]_w-\lambda\,[(\mathbf{n}\nabla)\,T]_w. \tag{9}$$

The boundary conditions for the longitudinal velocity, temperature and the characteristics of turbulence at the solid boundary are approximated by using the method of wall functions [6], according to which the diffusive flux of kinetic energy of turbulence through a solid boundary is assumed to be zero, and its rate of dissipation at a point P, located at distance y_r from the boundary, is given by the formula

$$\varepsilon_p = \left(\frac{3}{2}\right)^{0,5} \frac{C_\mu k_p^{1,5}}{\varkappa_2 y_p},$$

where $\kappa_2 = 0.19$ is an empirical constant, and the point P is chosen from the condition $30 < y_r^+ < 100$. The shear stress at the wall is computed from the formula

$$\tau_w = \frac{\varkappa C_\mu^{0,25} k_p^{0,5} u_p}{\ln(E y_p^+)},$$

which is also used in the finite-difference approximation of the diffusion terms in the Navier—Stokes equation at the wall. The temperature gradient at the wall is determined from the formula

$$\left(\frac{\partial T}{\partial y}\right)_w = \Pr \frac{y_p^+}{y_p} \frac{T_p - T_w}{F_p},$$

where the function F_r, following the recommendation of [6], is taken to be

$$F_p = \Pr_T \frac{\ln(E y_p^+)}{\varkappa} + 9{,}24 \Pr\left(\frac{\Pr}{\Pr_T} - 1\right)\left(\frac{\Pr_T}{\Pr}\right)^{0,25}.$$

Method of Numerical Solution. The system of differential equations of the method of discrete ordinates, Eqs. (1) and (2), along with the energy equation (3) is solved by a finite-difference method. To obtain the discrete analog of Eq. (3) we used the method of integration over a control volume [8]. During testing of the program it was established that for thermal fluxes typical of the furnaces of tubular ovens an explicit scheme for simultaneous integration of the equations of radiative transfer and energy becomes unstable. To avoid this instability one applies to the divergence of the radiative fluxes the lower relaxation

$$\operatorname{div} \mathbf{q}_p = \sigma \operatorname{div} \mathbf{q}_p^n + (1 - \sigma) \operatorname{div} \mathbf{q}_p^{n-1},$$

where the parameter $\sigma = 0.1$; and n is the number of iterations.

The well-known SIMPLE algorithm [8] was used to solve the system of differential equations (6) describing the field of turbulent flow of combustion products in the furnace chamber. Simultaneous solution of Eqs. (1), (3) and (6) with the corresponding uniqueness conditions was accomplished by the method of successive approximations. The above-described method of thermal design of furnaces of tubular ovens of box type was used in a group of applied programs, written in Fortran IV language for the ES computer.

Results of the Investigations. We investigated the influence of the width of the furnace chamber on external heat transfer in a tubular oven with free burning of the fuel (natural gas). The radiative heat transfer was examined in the S_2 approximation by the method of discrete ordinates ($N_0 = 4$), the radiative spectrum of combustion products was described in a six-band model [9], accounting for the bands at 1.5, 2.7, 6.3 and 10 μ of the H_2O radiation and at 2.7, 4.3, and 15 μ of CO_2. It was shown in [2] that in burning of gaseous fuel one can neglect radiative scattering at the individual particles, and therefore the calculations were done for $\beta = 0$. The dependence of the thermophysical properties of the combustion products on temperature was accounted for. Because there is a plane of symmetry only half of the furnace was considered (Fig. 1).

In free burning of the fuel a uniform distribution of the heat transfer regime was achieved. This regime is characterized by the fact that the tubular screen is not subject to the direct thermal action of the flame, since between the flame and the tubular screen there is a region of temperature that is low compared with the flame temperature. Since the absorption coefficient of H_2O and CO_2 is inversely proportional to temperature, this region acts as a thermal screen, blocking radiative heat transfer between the flame and the tubular screen. This is confirmed by the results of the computation shown in Figs. 2 and 3. In the flame region one observes maxima in the distributions of radiative and convective heat flux densities to the tubular screen. For a reduced width of furnace

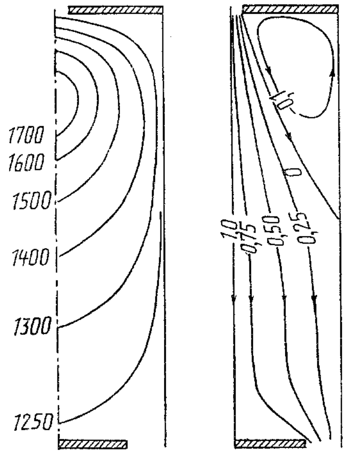

Fig. 2

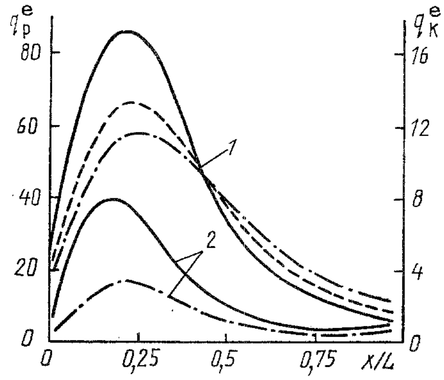

Fig. 3

Fig. 2. Isotherms (T, K) and stream lines (ψ/ψ_0, ψ_2 = 0.175 kg/(m·sec)) in a furnace for H = 2 m.

Fig. 3. Distributions of density of radiant heat flux (1) and convective heat flux (2) to a tubular screen; the solid lines are for H = 1.4; the broken lines are for H = 2 m; and the dot-dash lines are for H = 2.6 m.

TABLE 2. Influence of Furnace Width on Total Heat Transfer

H, m	$\bar{q}_p^e$, $\frac{kw}{m^2}$	$\bar{q}_K^e$, $\frac{kw}{m^2}$	T_{exit}, K
1,4	36,9	2,9	1221
2,0	33,8	2,6	1226
2,6	31,1	2,4	1230

chamber for maximum of q_r^e becomes more pronounced, and there is an increased level of nonuniformity of heating of the radiant tubes along the length and of the density of radiative heat flux to them in the flame region. The explanation is that as H is reduced there is less screening influence of the low temperature region on the flame. Here there is also a definite part played by the reverse flow zone, which elongates the low-temperature region of direct flow away from the tubular screen. For this reason in a wide furnace chamber below the flame region the combustion products have a higher temperature compared with a narrow furnace. This leads to the so-called inversion phenomenon. It can be seen in Fig. 3 that all three curves of the distribution of q_r^e intersect at one point, and below this point a wide furnace chamber has higher radiative heat flux density to the tubular screen. It should be noted that the location of the inversion point may appreciably affect the dependence of radiative heat transfer to the tubular screen on the furnace chamber width.

When the furnace chamber width is reduced the convective heat transfer is intensified, which can be explained by an increase of the mean rate of motion of the chimney gases. The position of the maximum in the distribution of q_K^e is influenced both by the aerodynamics of the furnace gases and by the temperature field. In a narrow furnace chamber the maximum in the distribution of q_K^e is located closer to the exit (Fig. 3).

Table 2 shows the computed data on total heat transfer. We obtained the result that a narrow furnace achieves a higher total heat transfer to the tubular screen. This result contradicts the data obtained in considering radiative heat transfer without computing radiative—convective interaction and the nature of the motion of the combustion products. With a width of furnace chamber of H = 1.4 m the total heat transfer was 16% higher compared with the case H = 2.6 m. The fraction of the convective contribution to the heat balance of the furnace depends weakly on H, and is 7% on the average.

Conclusion. Using the method suggested above we have investigated numerically the influence of furnace width on the external heat transfer in the furnace of a tubular oven of box type. It has been established that a narrow furnace chamber achieves higher total heat transfer to the tubular screen. However, then there is an increase of the degree of nonuniformity of heating of the radiative tubes along their length, and therefore the optimal width of the furnace chamber should be determined by considering the allowable values of temperature of the radiative tubes.

Notation. x, y) coordinates; ε) emissivity, or rate of dissipation of turbulent fluctuations; r) reflectance; β) scattering coefficient; $\alpha\lambda_k$) spectral integral over band k of the absorption coefficient; I_m^k) intensity of integrated radiation in the spectral interval $[\lambda_{k-1}, \lambda_k]$ in direction (μ_m, ξ_m); $J_{\lambda b}$) Planck function; N_0) number of chosen directions; $\Delta\lambda_k = \lambda_k - \lambda_{k-1}$) width of a spectral band; N_s) number of spectral bands; c_p) specific heat at constant pressure; ρ) density; u, v) components of the velocity vector; T) temperature; q_p^x, q_p^y) vector components of the radiative heat flux density; q_p; λ, μ) thermal conductivity and viscosity, respectively; $\mu_{ef} = \mu + \mu_T$; B_f) mass flow rate of fuel; Q_p^H) low heat of combustion of fuel; l_f) flame length; a) empirical constant; Γ_f) generalized transfer coefficient; S_f)source term; k) kinetic energy of turbulent fluctuations; P) pressure; R_μ) specific gas constant; Pr, Pr_T) molecular and turbulent Prandtl numbers; D) width of the entrance section; Tu) turbulence level at the entrance; $(T_{ext} - T_w)$ temperature drop across the claque; R_f) thermal resistance of the claque; n) interior normal to the boundary; ∇) nabla operator; y_p^+) dimensionless distance; E = 8.8) empirical constant; κ) Karman constant; q_p^e, q_K^e) densities of radiative and convective heat flux to the tubular screen; $\bar{q}_p^e$, $\bar{q}_K^e$) values of q_p^e, q_K^e averaged over the length of the radiant tubes; H) furnace width; $g = 9.81$ m/sec^2; T_{exit}) temperature of gases at the furnace exit; ψ) stream function; L) height of the furnace chamber. Subscripts: w) value at the boundary; o) value at the furnace entrance; r) value at distance y_r from the boundary; t) value of the characteristic of turbulence.

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